## Sun Dogs and Halos

Recently a spectacular halo was visible around the Moon from most parts of the UK. Unfortunately I missed it but it inspired me to look into how halos, sun dogs and other atmospheric phenomena occur. (Incidentally, the planet next to the Moon in the photograph below is Jupiter. The photo was taken by Jonny Gios on $25^{\text {th }}$ November 2023)

$22^{\circ}$ Moon halo over Scout Scar
Let's start with a more familiar phenomenon - the rainbow.

## Rainbows

When sunlight strikes a spherical raindrop, light is refracted when it enters the drop, then it is reflected once off the back of the drop and re-emerges having been refracted a second time. Depending on where the ray enters the drop, the angle at which it emerges can vary widely but it turns out that there is a maximum angle and that lots of rays which enter the drop in different places (highlighted in red in the image opposite) emerge at close to this maximum angle. For red light this angle is $42.4^{\circ}$ while for blue light (which bends more than red light) the angle actually turns out to be a bit smaller at $40.6^{\circ}$.
(If you are wondering why a lot of books talk about the angle of minimum deviation, they are talking about the complementary angles i.e. $137.6^{\circ}$ for red light and $139.4^{\circ}$ for blue light)

fig 1: Rays of light passing through a raindrop

Now when you turn away from the Sun and face the shower cloud, quite a lot of extra light is reflected from the backs of all the raindrops so the cloud looks quite bright. But beyond $42.4^{\circ}$, no light is reflected and the sky will look darker. The outer edge of the rainbow will be pure red and between $42.4^{\circ}$ and $40.6^{\circ}$ shades of mixed colour will appear tending towards greeny-blue. (Sometimes you can see more faint bands of red and blue inside the main rainbow. These are due to diffraction effects and I do not understand them.)


A particularly intense rainbow showing the brighter sky under the bow and also the secondary bow above it

The secondary bow is due to rays of light which reflect twice inside the drop. Most rays of light which bounce twice emerge from the other side without being deflected very much but, as with the primary bow, there is a cluster of rays which show a maximum deflection of about $130^{\circ}$. These form a 'bow' whose (bright) centre is the sun with a radius of $130^{\circ}$. Since this angle is greater than $90^{\circ}$ it is easiest to view by turning away from the Sun and pretending that is has a radius of $50^{\circ}$. Like the primary bow, the red colour is on the outside but, unlike the primary bow, the centre of the reflection is behind you so the colours are upside-down.

## The $22^{\circ}$ halo

The beautiful halo round the Moon illustrated earlier has a similar origin but is due to ice crystals in the stratosphere.

Everyone knows that snowflakes have hexagonal symmetry but the complex shape of a snowflake is a reflection of the many different environments which it passed though on its way to the ground. When conditions are ultra stable, large numbers of identical crystals may form the most important of which, from our point of view, are hexagonal prisms. If the prisms are longer than they are wide they are know as columnar crystals; if they are wider than they are long, they are called tabular crystals.

The $22^{\circ}$ halo is caused by a mass of randomly oriented crystals and the ray which is of interest is the one that enters one of the faces of the prism and exits from one of the two faces of the prism which are inclined at an angle of $60^{\circ}$. The angle by which a ray like this is deviated depends
on the angle at which it strikes the first face but, as with the raindrop, it turns out that there is a mninum angle of deviation and when rays of light strike the first face at all sorts of angles, most of them will be deviated through this special angle.

fig 2: A ray passing through a hexagonal prism

## Angle of minimum deviation


fig 3: A ray at minimum deviation
When a ray of light passes through a triangular prism it bends twice in the same direction. The total amount of bending (the 'deviation') depends on the angle of incidence at the first face ( $i$ ), the apex angle of the prism $(\alpha)$ and the refractive index of the material $(\mu)$. It turns out that the angle of deviation is a minimum when the ray passes through the prism symmetrically. This means that the angle of refraction at the first face is, in fact, $\alpha / 2$.

Using Snell's law for the refraction at the first face we have

$$
\begin{equation*}
\sin (i)=\mu \sin (\alpha / 2) \tag{1}
\end{equation*}
$$

where $r$ is the angle of refraction.
The total deviation $\theta$ is simply twice the deviation at the first face i.e.

$$
\begin{equation*}
\theta=2(i-\alpha / 2) \tag{2}
\end{equation*}
$$

The refractive index of ice is 1.31 and , being hexagonal, the ice crystals act like $60^{\circ}$ prisms. The angle of incidence at minimum deviation $i$ is therefore then angle whose $\sin$ is $1.31 \times \sin \left(30^{\circ}\right)$ which is $40.9^{\circ}$ and the deviation itself will be $2(40.9-30)=21.8^{\circ}$.

When rays of light strike a hexagonal ice crystal at all sorts of angles, therefore, most are deviated by more than $22^{\circ}$. But, just as with the rainbow, there is a large cluster of rays which deviate by this special angle which is why the sky appears brighter at this angle.

Since the refractive index of ice in red light is smaller than that for blue light, the angle of minimum deviation for red light will be smaller than that for blue. This may give the $22^{\circ}$ halo a reddish tinge on the inside of the halo but this is rarely seen as the halo is quite faint.

It is important to note that the $22^{\circ}$ halo is due to crystals which can be either tabular or columnar and that they are randomly oriented. This is why the $22^{\circ}$ halo is the most commonly seen kind of ice crystal halo.

## The $46^{\circ}$ halo

Although the majority of rays passing through a columnar crystal pass through faces inclined at $60^{\circ}$, a minority can pass through a pair uf faces inclined at $90^{\circ}$ e.g. by entering a side face and leaving through one of the ends (or vice versa of course). Under these circumstances, the effective angle of the prism is $90^{\circ}$ not $60^{\circ}$ and the angle of minimum deviation is $46^{\circ}$. The $46^{\circ}$ halo is reckoned to be six times fainter than the $22^{\circ}$ halo. It is almost always completely colourless.

## Sun dogs

The most common atmospheric phenomenon associated with ice crystals in the stratosphere is the sun dog. Unlike the $22^{\circ}$ and $46^{\circ}$ halos which are produced by randomly oriented crystals, sun dogs are produced by crystals whose longitudinal axis is aligned in the vertical direction. This comes about when relatively large tabular crystals fall vertically through the air. Aerodynamic forces tend to make them fall, like leaves, with the large flat surfaces at right angles to the direction of motion.

When the Sun is low down in the sky, rays of light passing through the plate, as in fig 2, have a minimum deviation of $22^{\circ}$ just as in the case of the $22^{\circ}$ halo. The difference in this case, however, is that all the crystals are oriented in the same direction and so the effect is very much brighter. In fact, the patch of light can be so bright that it can rival the brightness of the Sun itself. Naturally there are two $22^{\circ}$ sun dogs, one on either side of the Sun.

fig 4: Sun Dogs and the $22^{\circ}$ halo
Since blue light is deviated more than red light, the Sun Dogs appear blue on the outside and red on the inside - the exact opposite of a rainbow. This is because the Sun Dog depends on the angle being minimum whereas the rainbow depends on the angle being maximum.

Sun Dogs have their place in history. On a crisp clear February day in 1461 the young Yorkist prince Edward, soon to become Edward IV, faced a Lancastrian army on the slopes of Mortimer's Cross in Herefordshire. In Shakespeare's words: "Three glorious suns, each one a perfect sun; Not separated with the racking clouds, But sever'd in a pale clear-shining sky." appeared. Initially terrified of the sight, Edward rallied his troops by claiming that the sun dogs were in fact a sign that the battle would be won - as indeed it was.

The Sun Dogs shown in fig 4 coincide with the $22^{\circ}$ halo. As the Sun rises higher into the sky, the light rays enter the vertically oriented crystal at a more oblique angle. This has the effect of increasing the apex angle of the prism which, in turn, increases the angle of minimum deviation causing the Sun Dogs to move further away from the $22^{\circ}$ halo. This can be clearly seen in a remarkable photograph taken in 2018 in the North West Territories of Canada by Martin Male.

fig 5: Sun Dogs and halos in Canada 2018: Martin Male
The Sun is about $30^{\circ}$ above the horizon and the two Sun Dogs are well separated from the $22^{\circ}$ halo.

The other halos visible in this photograph will be explained later but the bright line which passes through Sun Dogs and the Sun itself is called the parhelic circle.

## The Parkelic Circle

Like the Sun Dogs the parhelic circle is caused by vertically aligned crystals whose faces act like tiny mirrors. Since any ray which bounces off a vertical surface does not change its angle with respect to the horizontal, all these points of light have the same altitude as the Sun. It can sometimes form a continuous circle round the whole sky like a line of latitude.

## The Tangent Arcs

The most conspicuous feature visible in the photo is what is called the upper tangent arc at the top of the $22^{\circ}$ halo. (There is a similar brightening of the halo beneath the Sun which is part of a lower tangent arc.)

As we have seen, the two Sun Dogs on either side of the Sun are caused by vertically oriented plate crystals. The upper and lower tangent arcs are caused by horizontally oriented columnar crystals whose longitudinal axis lies roughly at right angles to the line between the viewer and the Sun. These crystals bend light rays which would have passed over your head down into your eyes.

The shape of the 'wings' of the arc varies with the altitude of the Sun and is more difficult to
explain. Light from the 'wings' is due to light rays which pass obliquely through the crystal (thus increasing the angle of minimum deviation) and which also bounce off one of the end plates of the crystal (enabling the light to bend towards you as well as down).

fig 6: Tangent Arc with the Sun low on the horizon
This photo makes it clear that the tangent arcs are essentially vertical Sun Dogs.

## Other halos

This stunning picture taken by Michael Schneider in Switzerland shows a multitude of other optical phenomena. Note that the $22^{\circ}$ halo crosses in front of the mountains behind. It follows that these effects were due to ice crystals between the photographer and the mountains. In fact, they are probably due to minute crystals in the air immediately surrounding the photographer.

fig 7: Multiple optical phenomena

A key to all the effects is shown below.

fig 8: Multiple optical phenomena

## The Parry arc

Immediately above the upper tangent arc is a second arc labelled here 'Suncave Parry Arc'.
It will be recalled that the $22^{\circ}$ halo is formed by random orientations of hexagonal crystals and that the $22^{\circ}$ angle is the minimum deviation angle for a $60^{\circ}$ prism.

Parry arcs are formed by columnar crystals whose longitudinal axis is horizontal and whose top and bottom surfaces are also horizontal. This situation is known as the Parry orientation. They may also have an elongated cross section like this which may help them to adopt this seemingly unlikely state:


If the Sun is very low on the horizon, the initial ray will strike the first face horizontally and the angle of deviation of the ray will be $23^{\circ}$ but as the Sun rises higher in the sky the ray is deviated more and more and the Parry arc rises above the $22^{\circ}$ halo and becomes straighter. Above a certain elevation the arc bends the other way and becomes a 'sunvex' arc. (The excellent website atoptics.co.uk seems to imply that there are two arcs, one suncave and one sunvex but I have not come across a photograph which shows anything other than a suncave arc.)

## The circumzenithal arc

This arc is also also produced by Parry crystals whose top surface is horizontal. In the case of the circumzenithal arc the crystals are tabular in form. The rays of light enter the top surface at an angle of incidence equal to 90 minus the angle of elevation of the Sun and the effective apex angle of the prism is $90^{\circ}$. The best elevation angle turns out to be about $22^{\circ}$ (it is a pure coincidence that this is the size of the $22^{\circ}$ halo which is formed in a completely different way) and that the angle of deviation at this elevation is $46^{\circ}$. (Again, it is a complete coincidence that this is also the diameter of the $46^{\circ}$ halo. Both the $22^{\circ}$ halo and the $46^{\circ}$ halo are minimum deviation effects from randomly
oriented crystals and are therefore independent of the elevation of the Sun. The circumzenithal arc is caused by a unique ray which depends on the Sun's elevation.)

The ray then exits from one of the vertical faces of the hexagon opposite to the point of entry. Since this face may be be inclined $30^{\circ}$ either way, the ray may be skewed sideways and the angle of deviation will be greater than $46^{\circ}$. This turns the circumzenithal arc into a smily grin $68^{\circ}$ above the horizon as shown in the photo below which was taken in October 2012 in Boston, Massachusetts:

fig 9: The circumzenithal arc
Contrary to what is implied by the name, the arc only extends $10^{\circ}$ or $15^{\circ}$ on either side of the central line and does not completely encircle the zenith; nor is it strictly at the same elevation unlike the parhelic circle which can completely surround the viewer at exactly the same elevation all the way round.

Since the arc is caused by a unique ray passing through crystals all with the same orientation, the colours in the halo are much more saturated than the colours in a typical Sun Dog and, because blue light bends more than red light, the blue end of the spectrum is further from the Sun. When the Sun is at its optimal elevation, the circumzenithal arc can look like an inverted rainbow high in the sky and has caused many amateur observers intense puzzlement!

## The supralateral arc

Coinciding with the circumzenithal arc shown in fig 8 and also faintly visible in the above photograph is what is called the supralateral arc. This arc always coincides with the circumzenithal arc but, like the $46^{\circ}$ halo with which it is easily confused, it is centred on the Sun. Because its diameter depends on the elevation of the Sun, however, it must be caused by a unique ray passing through a pair of faces at right angles. While the circumzenithal arc is due to rays entering a tabular crystal whose longitudinal axis is vertical through the upper hexagonal face, the supralateral arc is formed by a ray which enters a horizontally oriented columnar crystal (i.e. a crystal in the Parry orientation) through one of its faces and exits through one of its ends (or vice versa).

fig 10: Rays of light through vertical and horizontal (Parry) crystals

Rotating the tabular crystal about its vertical axis will only cause the deviated ray to bend sideways slightly. Likewise, rotating the columnar crystal about a vertical axis will have exactly the same effect. Rotating the columnar crystal about its longitudinal (horizontal) axis, however, has a completely different effect. Now the top and bottom surfaces are no longer horizontal. Nevertheless, all those crystals whose longitudinal axis happens to be parallel to the line between the viewer and the Sun can bend light towards the viewer if the one of its surfaces happens to be at just the right angle too.

While it may seem unlikely that there would be a sufficient number of crystals which are aligned in just the correct way, as the millions of crystals tumble about, every one which passes through the critical orientation will emit a tiny flash of light which, overall, can amount to faint but visible halo.

Naturally similarly oriented crystals below the Sun can deviate light upwards producing a second arc called the infralateral arc but this is rarely seen for obvious reasons.

The following photograph also clearly shows four rainbow coloured highlights on a circle surrounding the Sun with the same radius as the supralateral arc. In fact, if we include the upper and (invisible) lower lateral arcs, it is clear that there are six highlights in the shape of a hexagon. These must be due to rays of light which are both deviated by the passage through the $90^{\circ}$ prism and reflected internally from one of the inclined faces of the crystal. As far as I know, these highlights do not appear to have a name. This is surprising as they are very prominent in figs 5 and 7 as well as in the photo below.

fig 11: A particularly fine supralateral and infralateral arc

## Other arcs

Two other arcs are labelled on fig 8, the Moilanen arc and the helic arc. The origin (and even existence) of these arcs is disputed.
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